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Question Paper Code : 60769

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS – I

(Common to all branches)

(Regulations 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue.

2. Discuss the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.

3. Find the equation to the sphere, having the points (–4, 5, 1) and (4, 1, 7) as ends of a diameter.

4. Prove that $9x^2 + 9y^2 - 4z^2 + 12yz - 6zx + 54z - 81 = 0$ represents a cone.

5. Find the radius of curvature of the curve given by $y = c \log \sec \frac{x}{c}$.

6. Find the envelope of the family of lines $y = mx + \frac{a}{m}$, where m is the parameter and a is a constant.

7. If $u = f(y - z, z - x, x - y)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

8. If $r = \frac{yz}{x}, s = \frac{zx}{y}, t = \frac{xy}{z}$, find $\frac{\partial(r,s,t)}{\partial(x,y,z)}$.

9. Express $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\, dx\, dy$ into polar coordinates.

10. Evaluate : $\int_0^2 \int_0^y \int_0^x dx\, dy\, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. (8)

(ii) Verify the Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and hence find A^{-1}. (8)

Or

(b) Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ into a canonical form by an orthogonal transformation and hence find its nature. (16)

12. (a) (i) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 + 2x - 2y + 4z - 19 = 0$ and $x + 2y + 2z + 7 = 0$. (8)

(ii) Find the equation of the cone whose vertex is the point $(1, 1, 0)$ and whose base in the curve $y = 0$, $x^2 + z^2 = 4$. (8)

Or

(b) (i) Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. (8)

(ii) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$. (8)

13. (a) (i) Find the envelope of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters are related by the equation $a^2 + b^2 = c^2$. (8)

(ii) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. (8)

Or

(b) (i) Find the radius of curvature and centre of curvature of the parabola $y^2 = 4ax$ at the point t. Also find the equation of the evolute. (10)

(ii) Find the envelope of the circles drawn upon the radius vectors of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as diameter. (6)

14. (a) (i) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \tan u$. (8)

(ii) Find the extreme values of $f(x,y) = xy(a-x-y)$. (8)

Or

(b) (i) Expand $e^x \cos y$ in powers of x, y upto the second degree terms using Taylor's theorem. (8)

(ii) Find the greatest and least distances of the point (3, 4, 12) from the unit sphere whose centre is at the origin. (8)

15. (a) (i) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ and hence evaluate it. (8)

(ii) Find the area that lies outside the circle $r=2\cos\theta$ and inside the circle $r=6\cos\theta$, using double integration. (8)

Or

(b) (i) Find the volume of the cylinder $x^2+y^2=25$ bounded by the planes $z=1$ and $x+z=10$. (8)

(ii) Evaluate $\iint_R \frac{xy\, dx\, dy}{\sqrt{x^2+y^2}}$, where R is the region in the first quadrant enclosed by the circles $x^2+y^2=4$ and $x^2+y^2=16$. (8)
